

# Effect of Coefficient of Performance for Estimation of Optimum Wind Turbine Generator Parameters

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**Abstract-**This paper presents estimation of optimum speed parameters of wind turbine generator using new formulation of capacity factor. The formulation includes the effect coefficient of performance between cut-in and rated wind speed of the wind turbine generator. Second Mean Value Theorem for integrals and Weighted Average Harmonic Mean are applied to derive the capacity factor expression. The proposed model is tested using the actual wind data measured at Kappadgudda wind site. Analytically computed speed parameters are compared with the speed parameters of wind turbine generator installed at site to show the effect of coefficient of performance. It is observed from results that the accuracy in assessment of power in the wind and performance of wind turbine generator improves by 20 % with the new model. The proposed model helps in wind farm planning and site matching of wind turbine generator.

**Keywords-** Coefficient of Performance; Capacity Factor; Wind Turbine; Power Curve

## I. INTRODUCTION

The average electrical power output depends upon wind velocities, turbine speed characteristics cut-in ( $V_c$ ), rated ( $V_r$ ), furling ( $V_f$ ) and aerodynamic design of rotor blades. Capacity Factor (CF), tip speed ratio and Coefficient of Performance ( $C_p$ ) are key factors that govern the optimum design of rotor blades and speed parameters of wind turbine generator (WTG). In order to estimate the performance of a WTG, appropriate modeling of CF considering aerodynamic analysis of blade and design parameter optimization for wind turbine is very vital. Aerodynamic design of wind turbine blades is dictated by  $C_p$  because  $C_p$  relates the wind speed, available mechanical power at the shaft and combines all the essential aerodynamic properties of a wind turbine. Further  $C_p$  is a turbine specific data and is normally provided by the manufacturer, during manufacturing of wind turbine, if accurate assessment of  $C_p$  is not considered energy will be lost during the high wind velocities.

Researchers have modeled CF to estimate average power production and performance evaluation of WTG<sup>[1-4]</sup> assuming the value of  $C_p$  is constant between  $V_c$  to  $V_r$ . The annual and monthly CF has been statistically computed<sup>[1]</sup> with  $C_p$  as a constant value. Modeling of CF in estimation of optimum speed parameters of wind turbine using normalized power curve<sup>[2]</sup> with assumption that  $C_p$  equal to  $C_{pr}$ . Further variation of  $C_p$  is taken in account while modeling CF<sup>[4-7]</sup>, but  $C_p$  is taken as  $C_{pmax}$  between  $V_c$  and  $V_r$ . However,  $C_p$  of wind turbine is neither constant nor maximum between  $V_c$  and  $V_r$  and it varies as function of tip

speed ratio as shown in Fig. 1<sup>[4]</sup>. In view of estimating higher energy production at higher capacity factor, this paper presents a derivation of an expression for CF considering the effect of aerodynamic properties of wind turbine blade indicated by  $C_p$ .

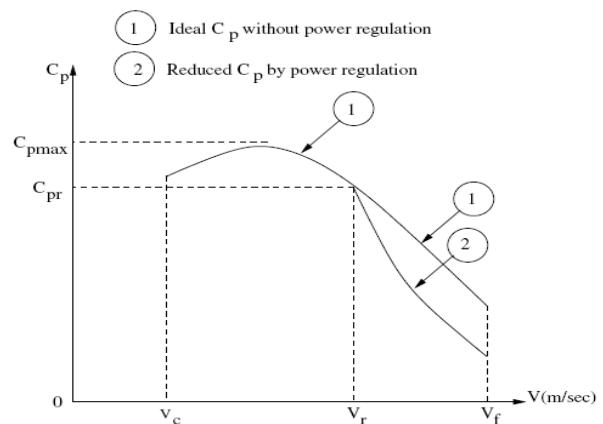


Fig. 1 Variation of  $C_p$  of WTG System

## II. PROPOSED MODEL

The electrical power output of generator  $P_e$  is given by the product of mechanical power output and the generator efficiency  $\eta_g$ <sup>[1]</sup>

$$P_e = \frac{1}{2} \eta_m \eta_g \rho A C_p (\lambda, \beta) v^3 \quad (1)$$

where

$\rho$ : air density (typically 1.225 kg/m<sup>3</sup> at sea level with standard conditions

$A$  : area swept by the rotor blades

$v$  : wind speed

$\eta_m, \eta_g$  : mechanical and generator efficiency

$\eta_m, \eta_g$ : coefficient of performance of the wind turbine is a function of tip speed ratio  $\lambda$  and blade pitch angle  $\beta$ .

Aerodynamic efficiency  $C_p$  of wind turbine is the ratio of turbine power to wind power<sup>[5]</sup>. The power coefficient  $C_p$  is given in terms of the blade pitch angle  $\beta$  and the tip-speed ratio  $\lambda$ . The assumption of linear torque versus speed characteristics are not valid for wind turbines, which are not operated over wind range of wind speeds<sup>[5]</sup>. Analytical analysis of aerodynamic system of such type of wind turbine is more complex. Consequently, numerical approximations have been developed in order to calculate the mechanical power characteristic of the wind turbine and a bi-dimensional characteristic function of  $C_p$  has been used and

validated in the laboratory [7]. Further to simplify the approximation of  $C_p$ , this paper considers blade pitch angle to be zero and is represented by non linear function as

$$C_p = 0.244 \left( \frac{130}{\lambda} - 6.56 \right) e^{-13.3/\lambda} \quad (2)$$

$$\lambda = \frac{\omega r}{v} \quad (3)$$

where  $\omega$ =rotational speed (rad/s),  $r$ =radius of rotor in m.

CF is defined as the ratio of average electrical power output to the rated electrical power output [2].

$$CF = \frac{1}{V_r^3 C_{pr}} \int_{V_c}^{V_f} C_p V^3 f(v) dv + \int_{V_r}^{V_f} f(v) dv \quad (4)$$

, where  $C_{pr}$ = Coefficient of performance at rated wind velocity,  $f(v)$  is the Weibull probability density function.

Power extraction is higher, if variation of  $C_p$  is taken into account during modeling of CF as compared to the constant  $C_p$  as shown in Fig. 2.

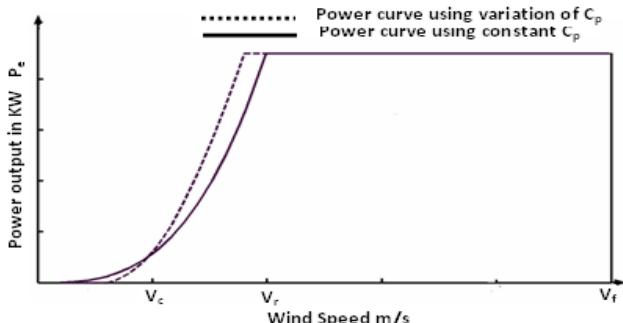


Fig. 2  $P_e$ versus wind Speed

Substituting (2) in (4), the expression for CF can be written as:

$$CF = \overbrace{\frac{31.72}{V_r^3 C_{pr} \omega r} \int_{V_c}^{V_f} e^{-13.3V/\omega r} v^4 f(v) dv}^a - \overbrace{\frac{1.6064}{V_r^3 C_{pr}} \int_{V_c}^{V_f} e^{-13.3V/\omega r} v^3 f(v) dv}^b + \overbrace{\int_{V_r}^{V_f} f(v) dv}^c \quad (5)$$

Equation (5) contains three integrals and is solved using SMVT [9]. SMVT is applied to first integral of (5) to obtain extreme functions by substituting  $V_r$  and  $V_c$  in (6).

$$CF = \frac{63.44 V_r e^{-13.3(V_r+V_c)/\omega r}}{\omega r C_{pr} (e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r})} \left[ P^4 e^{-(V_c/c)^k} - e^{-(V_r/c)^k} + \frac{4\Gamma\left(\frac{4}{k}\right)}{k\left(\frac{V_r}{c}\right)^4} \left\{ \gamma\left(\left(\frac{V_r}{c}\right)^k, \left(\frac{4}{k}\right)\right) - \gamma\left(\left(\frac{V_c}{c}\right)^k, \frac{4}{k}\right) \right\} \right] \\ - \frac{1.6064 e^{-13.3(V_r+V_c)/\omega r}}{C_{pr} (e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r})} \left[ P^3 e^{-(V_c/c)^k} - e^{-(V_r/c)^k} + \frac{3\Gamma\left(\frac{3}{k}\right)}{k\left(\frac{V_r}{c}\right)^3} \left\{ \gamma\left(\left(\frac{V_r}{c}\right)^k, \left(\frac{3}{k}\right)\right) - \gamma\left(\left(\frac{V_c}{c}\right)^k, \frac{3}{k}\right) \right\} \right] \\ + e^{-(V_r/c)^k} - e^{-(V_f/c)^k} \quad (13)$$

where  $V_r/c$ : Normalized rated speed,  $\Gamma$ : Gamma function  $\gamma$ : incomplete Gamma function.

$$a = \frac{31.72}{V_r^3 C_{pr} \omega r} \int_{V_c}^{V_f} e^{-13.3V/\omega r} v^4 f(v) dv \quad (6)$$

$$f(v_r) = e^{-13.3V_r/\omega r}, \quad f(v_c) = e^{-13.3V_c/\omega r}$$

Weighted average for SMVT can be determined using Arithmetic Mean (AM), Geometric Mean (GM) and Harmonic Mean (HM), however in the paper HM is considered as

- It is based on all observation
- It is not affected very much by fluctuations of sampling.
- It is rigidly defined
- It is particularly useful in averaging special types of rates and ratios where time factor is varying.

$$HM = \frac{2e^{-13.3(V_c+V_r)/\omega r}}{e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r}} \quad (7)$$

Substituting  $HM$  in first and second integral of (5) corresponding expressions are given by

$$a = \frac{31.72 HM}{V_r^3 C_{pr} \omega r} \int_{V_c}^{V_f} v^4 f(v) dv \quad (8)$$

$$b = \frac{1.6064 HM}{V_r^3 C_{pr}} \int_{V_c}^{V_f} v^3 f(v) dv \quad (10)$$

$$c = \int_{V_r}^{V_f} f(v) dv \quad (11)$$

Substituting (a), (b) and (c) in (5), modified expression of the CF is given.

$$CF = \frac{31.72 HM}{V_r^3 C_{pr} \omega r} \int_{V_c}^{V_f} v^4 f(v) dv - \frac{1.6064 HM}{V_r^3 C_{pr}} \int_{V_c}^{V_f} v^3 f(v) dv + \int_{V_r}^{V_f} f(v) dv \quad (12)$$

Further expression (12) is solved using integration by parts [2] detailed derivation is given in Appendix. New formulation for CF is given by.

$$CF = \frac{63.44 V_r e^{-13.3(V_r+V_c)/\omega r}}{\omega r C_{pr} (e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r})} \left[ P^4 e^{-(V_c/c)^k} - e^{-(V_r/c)^k} + \frac{4\Gamma\left(\frac{4}{k}\right)}{k\left(\frac{V_r}{c}\right)^4} \left\{ \gamma\left(\left(\frac{V_r}{c}\right)^k, \left(\frac{4}{k}\right)\right) - \gamma\left(\left(\frac{V_c}{c}\right)^k, \frac{4}{k}\right) \right\} \right] \\ - \frac{1.6064 e^{-13.3(V_r+V_c)/\omega r}}{C_{pr} (e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r})} \left[ P^3 e^{-(V_c/c)^k} - e^{-(V_r/c)^k} + \frac{3\Gamma\left(\frac{3}{k}\right)}{k\left(\frac{V_r}{c}\right)^3} \left\{ \gamma\left(\left(\frac{V_r}{c}\right)^k, \left(\frac{3}{k}\right)\right) - \gamma\left(\left(\frac{V_c}{c}\right)^k, \frac{3}{k}\right) \right\} \right]$$

In Equation (13) CF is expressed in terms of aerodynamic properties of blade and normalized rated wind velocity. First two terms in the expression of CF contains weighted  $HM$ , which incorporates the variation of  $C_p$  into

the CF. Normalized average power and Turbine Performance Index (TPI) curve are defined as [2].

$$P_N = \frac{P_{eavg}}{\eta_{mr} \eta_{gr} (\rho/2) A c^3} = CF \left( \frac{V_r}{c} \right)^3 \quad (14)$$

$$TPI = \frac{P_N * CF}{P_{N,max} * CF_{max}} \quad (15)$$

CF and  $P_N$  are computed using (13) and (14) for different values of  $V_r/c$  for known values of Weibull parameter. The normalized rated speed,  $V_r/c$  can be obtained corresponding to any point on the TPI curve. The value of rated speed  $V_r$  for the site with known scale factor at  $TPI_{max}$  becomes the optimum rated wind speed. Other two turbine parameters, namely cut-in and cut-out velocities, are estimated using the relations  $V_c = pV_r$  and  $V_f = qV_r$ , where  $p < 1$  and  $q > 1$ . Typical values of  $p$  and  $q$  for Indian sites are found to be **0.275** and **1.85**, respectively [2].

### III. RESULTS AND DISCUSSIONS

Proposed CF model is validated using two case studies. SMVT is used to incorporate the variation of  $C_p$  into the CF.

Monthly CFs are obtained from new formulation of CF using Cubic Mean Cube root (CMC) wind velocities. Normalized power, CF and TPI curves are drawn for the site whose Weibull shape  $k$  and scale  $c$  parameters known. Speed parameters for wind turbines are estimated and compared with installed turbine at site.

*Case Study-1:* Kappadgudda wind power station, Karnataka state, India, is used for this case study. Analytically computed values of monthly and annual CF using proposed model, existing method [2] and actual capacity factors measured at Kappadgudda site are given in Table I and Fig. 3. The optimum turbine performance index curve, drawn for annual  $k=2.3505$  and  $c=9.8054$  for wind site is shown in Fig. 5. The speed parameters derived from Fig. 4 are given in Table II.

*Case Study-2:* Purpose of this case study is to validate the proposed model for the second set of wind data collected in the year 2007 from Kappadgudda wind site. Analytically obtained monthly and annual CFs with variations of  $C_p$ , without  $C_p$  and actual CFs are given in Table I and Fig. 4. Speed parameters of wind turbine for the site are derived from Fig. 6, given in Table II.

Table I COMPARISON OF CF WITH AND WITHOUT  $C_p$  AT KAPPADGUDDA WIND SITE

Data for the year 1998							Data for the year 2007							
Month	CMC m/s	$C_p$	c	k	Capacity Factor			CMC m/s	$C_p$	c	k	Capacity Factor		
					With $C_p$	Without $C_p$	Actual					With $C_p$	Without $C_p$	Actual
Jan	5.7955	0.4006	6.4796	3.1085	0.1067	0.1073	0.1377	6.5712	0.4342	7.309	3.447	0.158	0.1680	0.2307
Feb	6.3556	0.4271	7.1663	2.4533	0.2014	0.1629	0.1915	6.7066	0.4378	7.511	2.998	0.186	0.1900	0.2414
March	6.0523	0.4143	6.8338	2.2102	0.1855	0.1513	0.1710	5.8867	0.4058	6.644	2.322	0.143	0.1502	0.1973
April	7.8349	0.4480	8.8468	2.1894	0.3774	0.2928	0.1869	7.4274	0.4480	8.386	2.251	0.296	0.2801	0.2254
May	10.9003	0.3764	12.2484	2.7570	0.6731	0.5408	0.4060	9.9361	0.4076	11.121	3.032	0.582	0.4932	0.5284
June	10.4821	0.3905	11.6194	3.6749	0.6982	0.5194	0.5410	10.5936	0.3868	11.940	2.481	0.620	0.5361	0.6543
July	11.7628	0.3460	13.1139	3.2969	0.7726	0.6225	0.5732	11.1845	0.3666	12.600	2.533	0.643	0.5702	0.6965
Aug	10.4108	0.3928	11.5997	3.3358	0.6736	0.5107	0.4592	10.3815	0.3938	11.570	3.301	0.637	0.5434	0.6900
Sept	8.2827	0.4444	9.3133	2.7045	0.4235	0.3170	0.3874	9.3345	0.4242	10.400	3.352	0.523	0.4461	0.5510
Oct	5.6524	0.3916	6.3811	2.2725	0.1408	0.1215	0.2319	6.6055	0.4351	7.455	2.334	0.215	0.2012	0.2123
Nov	7.5004	0.4482	8.3230	3.6044	0.2854	0.2193	0.1497	6.8428	0.4408	7.363	3.234	0.189	0.1945	0.2035
Dec	6.6385	0.4360	7.4414	2.9325	0.2043	0.1673	0.1858	7.1612	0.4458	8.049	2.741	0.254	0.2301	0.2693
Annual	8.6892	0.4097	9.8054	2.3505	0.4643	0.3601	0.3018	6.5712	0.4342	7.309	3.447	0.473	0.4152	0.4485

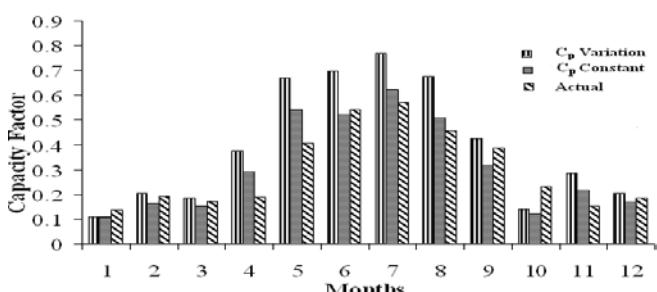


Fig. 3 CF with  $C_p$  variation and constant  $C_p$  for Vestas RRB at Kappadgudda

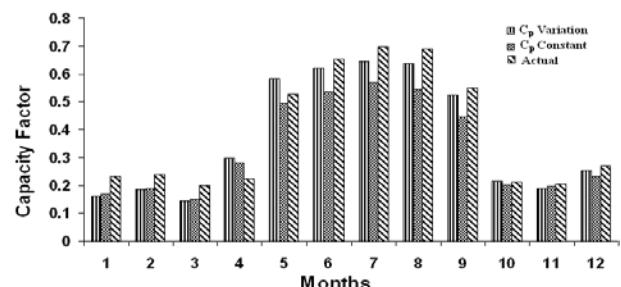


Fig. 4 CF with  $C_p$  variation and constant  $C_p$  for Enercon at Kappadgudda

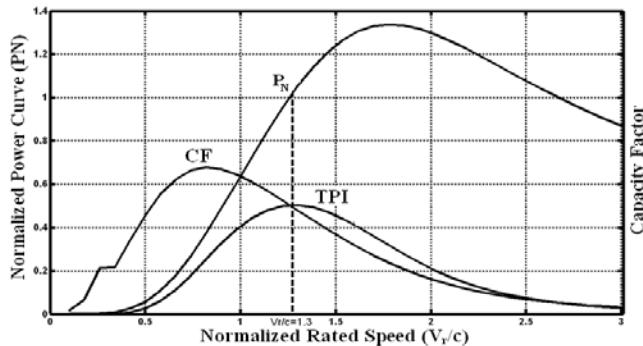


Fig 5: Turbine Performance Index for Kappadgudda site (1998 data)

Table II Turbine Speed Parameters obtained from Fig. 4 and Fig.5

Vestas-RRB turbine, D=30m (Weibull parameters at site k=2.3505, c=9.8054, for 1998 data, $\omega_{\text{rated}}=43 \text{ rad}$ )			Enercon E-30, D = 30 m (Weibull parameter at site k=2.469, c=10.170 for 2007 data, $\omega_{\text{rated}}=43 \text{ rad}$ )		
Parameter	Variation of $C_p$	Constant $C_p$	Actual	Variation of $C_p$	Constant $C_p$
$V_c$	3.50	3.2	3.5	3.41	3.30
$V_r$	12.75	11.7	13.5	12.42	11.39
$V_f$	23.58	21.7	25.0	22.98	21.01
CF	0.47	0.36	0.30	0.498	0.415
$P_N$	1.05	0.80	0.94	0.950	0.780
					0.920

**Discussions:** It is very interesting to note that, the analytical monthly and annual CF obtained from proposed model with  $C_p$  are higher than the existing model<sup>[2]</sup> by 23 % and 17 % during 1998 and 2007 respectively. Further it is observed that from the first case study that,  $V_r/c = 1.30$  at which PIC is maximum,  $CF$  and  $P_N$  normalized power from proposed model is 0.42 and 1.05 respectively. These parameters obtained are higher than existing methods and actual wind turbine installed at the site. Similarly for the second case study, at PIC maximum, normalized rated speed is  $V_r/c = 1.24$  shown in Fig. 6. It is observed from Table II that, marginal variations of CF,  $P_N$  and speed parameters between proposed method and actual wind turbine installed at site. However variation is more between proposed and existing method. Finally it is concluded that speed parameters estimated for wind turbine from proposed model will yield higher energy at higher capacity factor for both study years.

#### IV. CONCLUSION

Estimation of optimum speed parameter of wind turbine using new formulation of capacity factor is presented in the paper. New formulation of capacity factor is derived considering aerodynamic properties of wind turbine blade. Aerodynamic property of blade is governed by the  $C_p$  of wind turbine. SMVT and weighted HM are applied to considering the variation of  $C_p$  between  $V_c$  and  $V_r$  during modeling of CF. Estimated speed parameters from the proposed model are validated by comparing with existing method and speed parameters of Vestas RRB and Enercon E-30 wind turbine installed at the site. Conclusions derived from the two illustrative case studies are as follows.

- Power estimated is 20% higher by wind turbine using proposed formulation as compared to existing method.

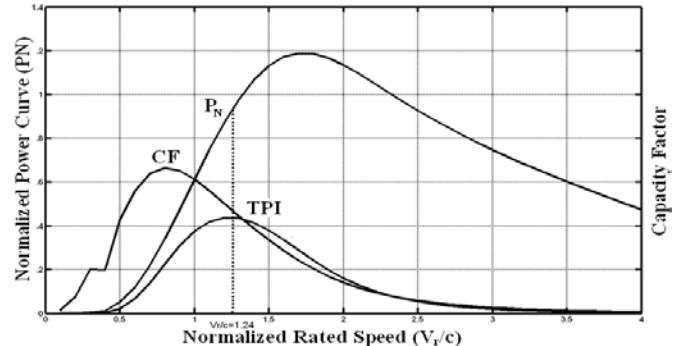


Fig 6: Turbine Performance Index for Kappadgudda site (2007 data)

- Speed parameters estimated for wind turbine taking into account the variation of  $C_p$  will yield higher energy at higher capacity factor.
- New formulation of capacity factor is a useful tool in site matching of wind turbine generators for wind potential site.

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## APPENDIX

## DERIVATION OF NEW FORMULATION OF CAPACITY FACTOR (CF) FOR WIND TURBINE

Capacity factor of WTG is given by [2]

$$CF = \frac{1}{V_r^3 C_{pr}} \int_{V_c}^{V_f} C_p v^3 f(v) dv + \int_{V_r}^{V_f} f(v) dv \quad A.1$$

Coefficient of Performance ( $C_p$ ) for WT is given by [5]

$$C_p = 0.244 \left( \frac{130}{\lambda} - 6.56 \right) e^{-13.3/\lambda} \quad A.2$$

Substituting (A.2) in (A.1), CF is obtained

$$CF = \frac{31.72}{V_r^3 C_{pr} wr} \int_{V_c}^{V_f} e^{-13.3V/wr} v^4 f(v) dv - \frac{1.6064}{V_r^3 C_{pr} V_c} \int_{V_r}^{V_f} e^{-13.3V/wr} v^3 f(v) dv \\ + \int_{V_r}^{V_f} f(v) dv \quad A.3$$

Second Mean Value Theorem (SMVT) for integral theorem states that

$$Y = \int_a^b f(x) \Phi(x) dx \quad A.4$$

$$Y = f(c) \int_a^b \Phi(x) dx \quad A.5$$

$f(c)$  is weighted Harmonic Mean (HM)<sup>[9]</sup> and is obtained by substituting upper and lower limits of (A.5)

$$f(c) = HM = \frac{2f(a)f(b)}{f(a) + f(b)} \quad A.6$$

SMVT is applied to (A.3) and weighted HM is given by

$$HM = \frac{2e^{-13.3(V_c+V_r)/\omega r}}{e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r}} \quad A.7$$

Substituting (A.7) in (A.3), CF is given by.

$$CF = \frac{63.44e^{-13.3(V_r+V_c)/\omega r}}{V_r^3 C_{pr} \omega r (e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r})} \int_{V_c}^{V_f} v^4 f(v) dv \\ - \frac{3.212e^{-13.3(V_c+V_r)/\omega r}}{V_r^3 C_{pr} (e^{-13.3V_r/\omega r} + e^{-13.3V_c/\omega r})} \int_{V_c}^{V_f} v^3 f(v) dv + \int_{V_r}^{V_f} f(v) dv \quad A.8$$

(A.8) contains three integral

$$I_1 = \frac{63.44e^{-13.3(V_r+V_c)/\omega r}}{V_r^3 C_{pr} \omega r (e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r})} \int_{V_c}^{V_f} v^4 f(v) dv, \quad A.9$$

$$I_2 = \frac{3.212e^{-13.3(V_c+V_r)/\omega r}}{V_r^3 C_{pr} (e^{-13.3V_r/\omega r} + e^{-13.3V_c/\omega r})} \int_{V_c}^{V_f} v^3 f(v) dv, \quad A.10$$

$$I_3 = \int_{V_r}^{V_f} f(v) dv$$

A.11

Substituting Weibull probability density function in (A.9-A.11)

$$f(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} e^{-\left( \frac{v}{c} \right)^k}$$

$$I_1 = \frac{63.44e^{-13.3(V_r+V_c)/\omega r}}{V_r^3 C_{pr} \omega r (e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r})} \int_{V_c}^{V_f} v^4 \frac{4k}{c} \left( \frac{v}{c} \right)^{k-1} e^{-\left( \frac{v}{c} \right)^k} dv$$

A.12

Using integration by substitution to (A.12) [2]

$$x = \left( \frac{v}{c} \right)^k, \quad \text{then} \quad dx = \left( \frac{k}{c} \right) \left( \frac{v}{c} \right)^{k-1} dv \quad \text{and}$$

$$v = cx^{1/k} \quad A.13$$

$$I_1 = \frac{ac^4}{V_r^3} \int_{V_c}^{V_f} x^{4/k} e^{-x} dx$$

$$\text{where: } a = \frac{63.44e^{-13.3(V_r+V_c)/\omega r}}{C_{pr} \omega r (e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r})}$$

$$I_1 = \frac{aV_r}{\left( \frac{V_r}{c} \right)^4} \int_{V_c}^{V_f} x^{4/k} e^{-x} dx \quad A.14$$

Applying integration by parts (A.14)

$$I_1 = \frac{63.44V_r e^{-13.3(V_r+V_c)/\omega r}}{\omega r C_{pr} (e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r})} \\ \left[ p^4 e^{-(V_c/c)^k} - e^{-(V_r/c)^k} + \frac{4\Gamma\left(\frac{4}{k}\right)}{k\left(\frac{V_r}{c}\right)^4} \left\{ \gamma\left(\left(\frac{V_r}{c}\right)^k, \left(\frac{4}{k}\right)\right) - \gamma\left(\left(\frac{V_c}{c}\right)^k, \frac{4}{k}\right) \right\} \right] \quad A.15$$

Similarly final expression is obtained for  $I_2$  and  $I_3$

$$I_2 = \frac{1.6064e^{-13.3(V_r+V_c)/\omega r}}{C_{pr} (e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r})} \\ \left[ p^3 e^{-(V_c/c)^k} - e^{-(V_r/c)^k} + \frac{3\Gamma\left(\frac{3}{k}\right)}{k\left(\frac{V_r}{c}\right)^3} \left\{ \gamma\left(\left(\frac{V_r}{c}\right)^k, \left(\frac{3}{k}\right)\right) - \gamma\left(\left(\frac{V_c}{c}\right)^k, \frac{3}{k}\right) \right\} \right] \quad A.16$$

$$I_3 = e^{-(V_r/c)^k} - e^{-(V_f/c)^k} \quad A.17$$

Combining (A.15), (A.16) and (A.17), we get

$$CF = \frac{63.44V_r e^{-13.3(V_r+V_c)/\omega r}}{\omega r C_{pr} (e^{-13.3V_c/\omega r} + e^{-13.3V_r/\omega r})}$$

$$\begin{aligned}
 & \left[ p^4 e^{-(V_c/c)^k} - e^{-(V_r/c)^k} + \frac{4\Gamma\left(\frac{4}{k}\right)}{k\left(\frac{V_r}{c}\right)^4} \left\{ \gamma\left(\left(\frac{V_r}{c}\right)^k, \frac{4}{k}\right) - \gamma\left(\left(\frac{V_c}{c}\right)^k, \frac{4}{k}\right) \right\} \right] \\
 & - \frac{1.6064e^{-13.3(V_r+V_c)/\alpha r}}{C_{pr}(e^{-13.3V_c/\alpha r} + e^{-13.3V_r/\alpha r})} \\
 & \left[ p^3 e^{-(V_c/c)^k} - e^{-(V_r/c)^k} + \frac{3\Gamma\left(\frac{3}{k}\right)}{k\left(\frac{V_r}{c}\right)^3} \left\{ \gamma\left(\left(\frac{V_r}{c}\right)^k, \frac{3}{k}\right) - \gamma\left(\left(\frac{V_c}{c}\right)^k, \frac{3}{k}\right) \right\} \right] \\
 & + e^{-(V_r/c)^k} - e^{-(V_f/c)^k}
 \end{aligned}$$

A.18



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